High Efficiency FEM Calculation of the Ionized Field under HVDC Transmission Lines

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Abstract — As many HVDC projects are constructed in China, the ionized field problem is widely concerned. Upstream finite element method (Upstream FEM) has good adaptive capacity, and was widely used to analyze the ionized field under HVDC lines. In order to obtain high calculation efficiency, Poisson equation direct solution method together with node reordering optimization and Cholesky decomposition is adopted. To solve Poisson equation repeatedly, the advantage is remarkable. Nodal charge density updating strategy is adopted to accelerate the convergence. These methods remarkably improve the caculation efficiency of Upstream FEM. For the 15,000 nodes scale ionized field problem of bi-polar 6 bundled UHVDC lines, the time cost was less than 5 seconds. And the algorithm shows good robustness to the charge density initial value in the numerical cases.

I. INTRODUCTION

High voltage direct current (HVDC) transmission system has many advantages over HVAC transmission system for long-distance and large bulk power transmission [1], so it has been developed rapidly in China. Two ±800kV UHVDC transmission systems have been put into operation in China, 2010.

Corona will occur at sufficiently high levels of conductor surface electric field [1]. Then the produced orientationally motional space charge (called ion current) will strengthen the static electric field produced by the conductor charge (called nominal electric field). The electric field produced by the conductor charge and space charge is called ionized field.

Many methods have been proposed to analyze the ionized field of the HVDC transmission lines [1]-[5]. In these methods, upstream finite element method (upstream FEM) has good adaptability, could consider the influence of wind, and was widely used [2], [4], [5]. But the calculation efficiency is not concerned very much. In [5], it costs 504 seconds to solve bi-polar 6 bundled HVDC lines ionized field problem with 3,716 nodes.

In this paper, to obtain high calculation efficiency of upstream FEM, direct solution method together with node reodering optimization and Cholesky decomposition is adopted to solve Poisson equation. Because of the particularity of the problem, Cholesky decomposition is calculated only once, and the Poisson equation solution procedure is changed as only a back substitution procedure in later iterations. The calculation is much more efficiency than Poisson equation iterative solution method as the equation is solved repeatedly. Node charge density updating order is important for the calculation efficiency, and an easy method is used to make nodal charge density updating order along the charge motional orientation to accelerate convergence. With above methods, for the 15,000 nodes scale ionized field problem of bi-polar 6 bundled UHVDC lines, the time cost was less than 5 seconds, only 1/100 of that in [5].

II. METHODOLOGY

A. Basic Equations

For bipolar HVDC transmission lines, the potential φ, the ionized field $\boldsymbol{E}_s$, and the positive and negative ion current density $\boldsymbol{j}_+$, $\boldsymbol{j}_-$ can be defined as follows [2].

$$\nabla^2\varphi = -(\rho_+ - \rho_-)/\varepsilon_0 \quad (1)$$

$$\boldsymbol{E}_s = -\nabla\varphi \quad (2)$$

$$\boldsymbol{j}_+ = \rho_+\boldsymbol{v}_+ \quad (3)$$

$$\boldsymbol{j}_- = \rho_-\boldsymbol{v}_- \quad (4)$$

$$\nabla\cdot\boldsymbol{j}_+ = -r\rho_+\rho_-/\mathrm{e} \quad (5)$$

$$\nabla\cdot\boldsymbol{j}_- = r\rho_+\rho_-/\mathrm{e} \quad (6)$$

Where ρ_+ and ρ_- are positive and negative space charge density, r is the ion recombination coefficient, e is the charge of the electron, $\boldsymbol{v}_+$ and $\boldsymbol{v}_-$ are velocity of ρ_+ and ρ_-, defined as follows.

$$\boldsymbol{v}_+ = K_+\boldsymbol{E}_s + \boldsymbol{w} \quad (7)$$

$$\boldsymbol{v}_- = -K_-\boldsymbol{E}_s + \boldsymbol{w} \quad (8)$$

Where $\boldsymbol{w}$ is the wind velocity vector, K_+ and K_- are the positive and negative ion mobility respectively.

B. High Efficiency Solution of Poisson Equation

Coupled of (1)-(8), the governing equations are nonlinear, and iterative calculation method is adopted. Upstream FEM has good adaptability in 2D ionized field calculation [2], [4], [5]. First, it's assumed that space charge density is some initial value and node potential is calculated by Poisson equation (1). Then according to the electric field, new space charge density is calculated by charge continuity equations (5). Next, potential is calculated by (1) and so on, through to all the space charge density and electric field satisfies the original differential equations and boundary condition with a certain error range. The solution time of (1) is the chief part of the total solution time, its solution method would be investigated next.

Matrix equation transformed from (1) is shown as

$$\begin{cases} \boldsymbol{S\Phi} = \boldsymbol{F} \\ \varphi_i = u \qquad\qquad i \in \partial\Omega \end{cases} . \quad (9)$$

After all the enforcement components are substituted into, the Matrix equation is transformed as

$$\boldsymbol{S}'\boldsymbol{\Phi} = \boldsymbol{F}' . \quad (10)$$

From (10), it can be seen that $\boldsymbol{S}'$ is not changed in all the iterations, and direct solution method could be chosen. Cholesky decomposition of $\boldsymbol{S}'$ is needed only once at the beginning, after that solution of (10) every time is only a back substitution procedure.

For example, a 15,750 nodes ionized field problem, the number of nonzero elements in $\boldsymbol{S}'$ is 102,204. After Cholesky decomposition with symmetrical minimum degree method reordering, the number of nonzero elements of the lower triangular matrix is 448,721. If the iterations needed by ICCG are more than 4.5, this direct solution is faster. To reach the same precision, time cost by this method is much less than 1/10 of that by ICCG. This direct solution method is much faster than ICCG to solve (10) many times.

C. Nodal Charge Density Updating Order

In the calculation, nodal charge density is determined by charge density of the other two nodes attach to its upstream element, so the nodal charge density updating order has profound influence on the convergence speed. It is indicated that nodal charge density updating order should be outward from corona conductor, but without concrete implementation method in [2], [4]. The element charge updating order is very efficiency in [6] and could be spread to the situation of this paper. Without wind, positive charge updating order could be from the high potential to the low potential. Considering the negative charge density, the order is inversed.

Considering the wind, a virtual potential could be defined to describe the effect of wind and electric field. Nodal charge updating order could refer to the virtual potential, but now the positive charge density updating order is not just inversed to the negative charge density. The proposed method could keep nodal charge density updating order along the charge motional direction, could assure fast convergence, and is easy to implement.

III. VALIDITY AND APPLICATION

A. ±400kV HVDC Lines

In order to verify the validity of the algorithm, ionized field below a ±400kV HVDC lines is analyzed. The numerical result agrees with the measurement value [7] well as shown in Fig. 1. The height of the lines is 10.7 m, the lines have two 2×1.91 cm bundled conductors with the bundle spacing 45.7 cm. The spacing between the negative and positive lines is 12.2 m.

The calculation domain is discretized into 13,884 triangular elements, 7,196 nodes. If the node charge density initial value on the conductor surface is set as [5], the total calculation time is 1.46 seconds and if the nodal charge density initial value is 1/10 of that, the calculation time is 1.67 seconds. The calculation method has good robustness to the nodal charge density initial value.

B. ±800kV UHVDC Lines

Considering the ±800kV UHVDC lines, the height of the lines is 18 m, the lines have two 6×1.538 cm bundled conductors with the bundle spacing 0.45 m. The spacing between the negative and positive lines is 22 m. The ionized field below the lines is shown as Fig. 2.

The calculation domain is discretized into 30,382 triangular elements, 15,750 nodes. As the nodal charge density initial value on the conductor surface is set as [5], the total calculation time is 4.39 seconds and if the nodal charge density initial value is 1/10 of that, the calculation time is 4.92 seconds. The calculation method also has good robustness to the nodal charge density initial value. Compared with the similar structure HVDC lines numerical case with 3,716 nodes and time cost 504 seconds in [5], the calculation efficiency increased remarkably. The basic frequency of CPU used in this paper is 2.33GHz.

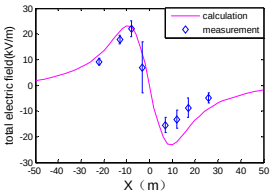

Fig. 1 Ionized field below ±400kV HVDC Lines

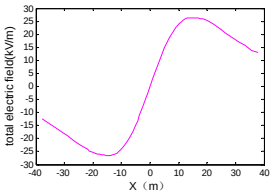

Fig. 2 Ionized field below ±800kV UHVDC Lines

IV. ACKNOWLEDGMENT

This project is supported by the key project of the National Natural Science Foundation of China (No. 51037001).

V REFERENCES

[1] M. P Sarma, “Corona performance of high-voltage transmission lines," Research studies Press LTD.. Baldock. Hertfordshire, England. 2000.pp. 3-22, 55, 179-213

[2] Takuma T，Ikeda T，kawamoto T. ”Calculation of ion flow fields of HVDC transmission lines by the finite element method”. IEEE Transactions on Power Apparatus and Systems, vol.100, no.12, pp. 4802-4810, 1981

[3] Z.M. Al-Hamouz, “Corona power loss, electric field, and current density profiles in bundled horizontal and vertical bipolar conductors,” IEEE Trans. on Industry Applications, vol. 38, no. 5, pp. 1182-1189, 2002.

[4] Tiebing Lu， Han Feng， Zhibin Zhao，et al. ”Analysis of the electric field and ion current density under ultra high-voltage direct-current transmission lines based on finite element method”. IEEE Transactions on Magnetics, vol. 43, no.4, pp.1221-1224, 2007

[5] Wei Li, Bo Zhang , Jinliang He. et al. "Ion flow field calculation of multi-circuit DC transmission lines". High Voltage Engineering, vol. 34, no. 12, pp. 2719-2725, 2008

[6] Long Zhengwei，Yao Qiang，Song Qiang，et al. "A second-order accurate finite volume method for the computation of electrical conditions inside a wire-plate electrostatic precipitator on unstructured meshes". Journal of Electrostatics，vol. 67, pp. 597-604, 2009.

[7] Johnson G. "Electrical fields and ion current s of a 400 kV HVDC test line". IEEE Transactions on Power Apparatus and Systems, vol. 102, no. 8. pp. 2559-2568. 1983